

# Hydraulic Fracturing and the Extraction of Minerals Through Wells

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## ABSTRACT

*"Hydraulic Fracturing" is a method used to create artificial fractures for the purpose of increasing flow capacity around a well or enhancing communication between two adjacent wells. The present paper reports on some theoretical, laboratory and field studies aimed at improving the knowledge of hydraulic fracture orientation and initiating pressure. Only vertical fractures are considered in detail, as they are the most abundant type encountered in the field.*

*The fractures are theoretically assumed to be tensile ruptures extending in a plane perpendicular to the direction of the smaller horizontal principal compressive stress. It is found that the pressures required to initiate and extend vertical fractures depend on the principal tectonic stresses, the porous-elastic parameters of the rock and its tensile strength.*

*Experimental work on simulated wells in laboratory rock samples under triaxial loading is described. Results confirm theoretical predictions of fracture type, fracture initiation pressure and fracture orientation.*

*In the field, oriented impression packers were used to determine the fracture azimuth at the well-bore. Results indicate that wells belonging to a common field yielded hydraulic fractures of approximately same orientation. This seems to substantiate the theoretical and laboratory conclusion that the smallest tectonic horizontal compressive stress direction determines the fracture orientation.*

## INTRODUCTION

Mining through wells is a common method used for the extraction of minerals like salt, potash, sulphur and especially petroleum. Sometimes the entire mining process necessitates one well only. Often it requires an injection well in addition to the production well. Whether the purpose is to increase flow capacity around one well, or to enhance communication between the injection and production wells, it is frequently necessary to induce artificial fractures in the ore bearing formation. The method usually employed is that of "hydraulic fracturing." It consists of sealing off a section of the well and pressurizing it by injecting in a "fracturing fluid" like water, brine, oil, etc. The pressure in the isolated interval is continuously raised until fracture occurs. The pressure then drops momentarily, but if pumping is continued vigorously the fracture opens up and propagates. Propping agents are sometimes introduced into the fracture to keep it from closing back when pumping is stopped. If the fracture is large enough and extends in the right direction, it can vastly increase production.

One of the unsolved problems of hydraulic fracturing is the ability to predict the inclination of the fracture plane and the direction it will follow. Knowledge of fracture orientation can be extremely valuable, for example, in the design of well layout in a producing field. Another problem, related mainly to the design of a fracturing job is the capability to foresee the maximum pressure

required to cause formation breakdown. The present report concerns itself mainly with these two latter problems. First it is shown that theoretically there is a close relationship between the state of stress in the earth and both the breakdown pressure and fracture direction. Then some laboratory tests on simulated wellbores are described. These tests seem to support the theoretical analysis. Finally a field method for detecting fracture direction is described, and a number of encouraging field results presented.

### THEORETICAL CONSIDERATIONS

The theory of elasticity of porous materials can be used to estimate the pressures required to initiate and extend hydraulic fractures and their orientation and direction. To do so, a theoretical model is constructed based on some limiting assumptions regarding the materials involved. It is assumed that the formation to be fractured is brittle elastic, porous\*, isotropic and homogeneous. The fluid flow through the formation is laminar and follows Darcy's Law. The state of stress in the formation, prior to drilling of wellbore, is generally non-hydrostatic with one of the principal stresses ( $S_{33}$ ) assumed to be acting in the vertical direction. The latter assumption is justified especially in formations of gentle dip (Anderson, 1963, p. 12). When a vertical circular wellbore is drilled the initial horizontal principal tectonic stresses ( $S_{11}$ ,  $S_{22}$ ) redistribute around the cylindrical cavity in a manner defined by Hirsch solution (Haimson, 1967, p. 311).

The pressurization of the open hole in the well generates two additional stress fields, one due to the radial pressure on the well wall, and the other due to fluid flow into the formation resulting from the difference between the well pressure ( $P_w$ ) and the reservoir fluid pressure ( $P_o$ ). The complete distribution of horizontal stresses around the wellbore is found by superposing the three mentioned stress fields (Haimson, 1967, p. 312). At the vertical wall of the open hole, and away from the hole ends, the most vulnerable stress is the tangential ( $S_{\theta\theta}$ ). Under normal in-situ stress conditions, this stress is the first to reach tensile values, as the wellbore pressure  $P_w$  rises, finally causing a vertical tensile rupture that originates at the well's wall. Looking at a cross section of the well, the fracture is most likely to initiate at two diametrically opposed points, whose connecting line is perpendicular to the larger tensile principal tectonic stress ( $S_{22}$ ), (Fig. 1). In terms of effective stresses  $\left[ \begin{matrix} \sigma_{ij} = S_{ij} \\ \sigma_{ij} = S_{ij} \end{matrix} \right.$

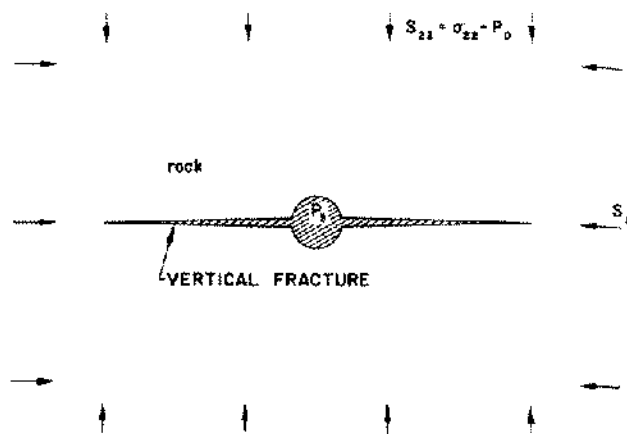


Figure 1. Cross section of vertically fractured well.

$$\left. \begin{matrix} P \text{ for } i = j \\ \text{for } i \neq j \end{matrix} \right\}$$

the tangential stress at these two points is given by

$$\sigma_{\theta\theta} = 3\sigma_{22} - \sigma_{11} + (2 - \alpha \frac{1 - 2\nu}{1 - \nu})(P_w - P_o) \quad (1)$$

where:

$\alpha$  = parameter of a porous elastic material; can be determined in the laboratory (Mann, 1960).

$$0 \leq \alpha \leq 1$$

$\nu$  = Poisson's ratio of the rock.

Failure in tension occurs when  $P_w$  reaches a critical value ( $P_c$ ), also called breakdown pressure, at which  $\sigma_{\theta\theta} \geq \sigma_t$ , where  $\sigma_t$  is the tensile strength of the rock in the horizontal plane. Hence the minimum critical pressure necessary to induce a vertical fracture is (Haimson, 1967, p. 312):

$$p_c^i - p_o = \frac{\sigma_t - 3\sigma_{22} + \sigma_{11}}{2 - \alpha \frac{1 - 2\nu}{1 - \nu}} \quad (2a)$$

where:

$$0 \leq \alpha \frac{1 - 2\nu}{1 - \nu} \leq 1$$

If the formation permeability to the fracturing fluid is negligible, the third mentioned stress field is zero and the critical pressure ( $p_c^i$ ) then becomes:

$$p_c^i - P_o = \sigma_t - 3\sigma_{22} + \sigma_{11} \quad (2b)$$

\* This includes nonporous formations as a special case.

From equations (2a) and (2b) it can be easily verified that  $P_c^i \geq P_c^p$ , or in other words, the breakdown pressure in a permeable formation is usually lower than the pressure required to fracture an impermeable but otherwise identical zone.

The rock parameters  $\sigma_1$ ,  $\nu$ ,  $\alpha$  can be measured in the laboratory in rock cores corresponding to the formation in question, and under conditions of loading and pore pressure similar to the in-situ ones.

It is assumed that the vertical tensile fracture initiated at the wellbore will extend along a plane perpendicular to the direction of the larger horizontal principal tectonic stress ( $S_{22}$ ). This assumption is based on the theory that a fracture follows the path of least resistance. The downhole pressure of the fracturing fluid necessary to keep the fracture open ( $P_s$ ) is the instantaneous shut-in pressure, and is given by:

$$P_s - P_o \geq -\sigma_{22} \quad (3)$$

It should be noted that the values of  $P_c$  and  $P_s$  are usually recorded by a pressure versus time plot taken during a fracturing job.

The case of horizontal fracturing initiation will not be considered here. Theoretical relationships and experimental results related to horizontal fractures can be found elsewhere (Haimson, 1968). There is, however, the possibility that fractures that are initiated in the vertical plane, due to the stress distribution at the wellbore, may change orientation and become horizontal away from the wellbore, so as to be perpendicular to the smallest compressive stress. Under normal in-situ stress conditions this possibility is rather remote, but when it occurs it is very hard to detect.

From equations (2, 3) it is evident that if the magnitudes of  $\sigma_{11}$  and  $\sigma_{22}$  are known, the breakdown pressure ( $P_c$ ) and the pressure required to keep the fracture open ( $P_s$ ) can be predicted. Moreover, it can be assumed that within a certain formation and depth, the tectonic stresses remain constant in an area which is undistributed geologically. It is expected therefore that in the same "neighborhood" of a producing field, wells will yield fractures oriented essentially parallel to each other. The magnitudes of  $P_c$  and  $P_s$  for these wells should not vary considerably from one well to the next.

In those locations where the two horizontal principal tectonic stresses are approximately equal ( $\sigma_{11} \approx \sigma_{22}$ ) there is no preferred direction for the vertical fracture and a weakness in the rock close to the wellbore can determine the fracture

orientation. Such a weakness may be in the form of a natural crack or an induced one (vertical notch). The orientation of fractures at the wellbore can be detected as described elsewhere in this report. If the fracture directions in a number of neighbor wells seem to be oriented at random, the horizontal state of stress is probably hydrostatic and by vertical notching fracture direction can actually be controlled.

## LABORATORY EXPERIMENTAL PROGRAM

In an attempt to verify the relationships outlined in the theoretical section between the orientation and breakdown pressure of vertical fractures and the magnitude and direction of the horizontal principal tectonic stresses, a series of tests were run on simulated wellbores in laboratory samples.

Rectangular rock specimens (5.0 inches  $\times$  5.0 inches  $\times$  5.5 inches), with a vertical central hole (.30 inch in diameter), were loaded triaxially in a specially built steel frame. By use of four flat-jacks mounted between the sides of the sample and the internal walls of the frame, two unequal and independent horizontal compression loadings were applied. The vertical loading was transmitted, through a specially built upper platen by a hydraulic compression tester. The upper platen also provided the fracturing fluid channeling into the internal hole of the samples (Haimson, 1968). The unequal external triaxial loading on the sample closely simulated the most general state of tectonic stresses in the earth. In those tests where no horizontal loading was applied, cylindrical samples were used (usually 6.0 inches high, 3.5–5.0 inches in diameter).

The simulated wellbore in the sample was an open hole, 2.0 inches long, terminated by the rock itself at one end and by a hollow metal plug at the other. Through this hollow plug, pressurized fracturing fluid was forced into the open hole.

To run a test, the predetermined external triaxial loading was first applied and kept constant throughout the rest of the experiment. Then the fracturing oil was introduced into the internal hole, pressurizing it at a constant rate (usually 6–15 psi/sec.). The pressure versus time curve was recorded in an X-Y plotter. At some critical (breakdown) pressure ( $P_c$ ), a sudden drop in pressure was observed, indicating fracture. The test was then stopped, the external loading removed and the sample observed, sectioned and photographed. The experimental data was recorded and checked against the theoretical predictions.

Different types of impermeable and permeable rock were tested. The natural rock was obtained from quarries throughout the country. The artificial rock was a mixture of water and gypsum cement (hydrostone), which when allowed to set formed a solid material of rock-like properties (Haimson, 1969).

The results of the laboratory tests can be summarized as follows:

1. All the hydraulic fractures obtained were tensile ruptures oriented either in the vertical or the horizontal plane, depending on the loading conditions.
2. In those cases where vertical fractures were obtained they were always perpendicular to the smaller horizontal compressive loading, notwithstanding the amount of oil penetration into the rock. Figure 2 shows a typical

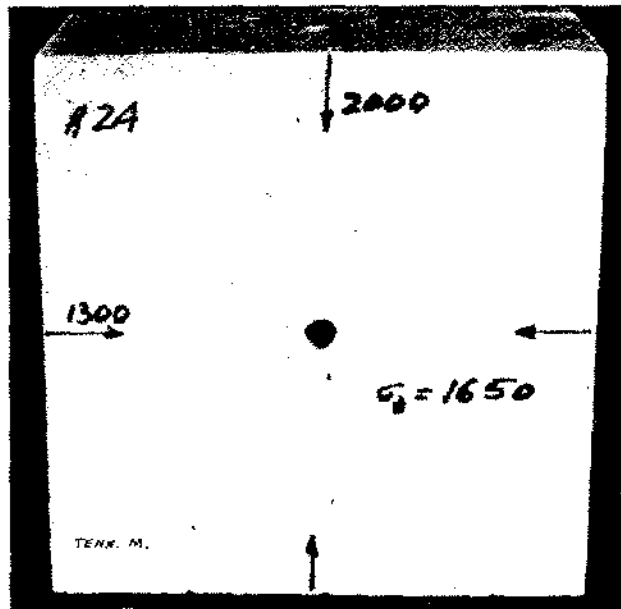


Figure 2. Vertical fracture in Tennessee Marble.

vertical fracture in impermeable Tennessee Marble. Figure 3 shows that the occurrence of a precrack in the impermeable charcoal granite sample did not interfere with the direction of the fracture normal to the smaller horizontal load. In permeable rock, like hydrostone and Berea Sandstone, shown in Figures 4, 5, fracturing fluid (hydraulic oil) leak-off

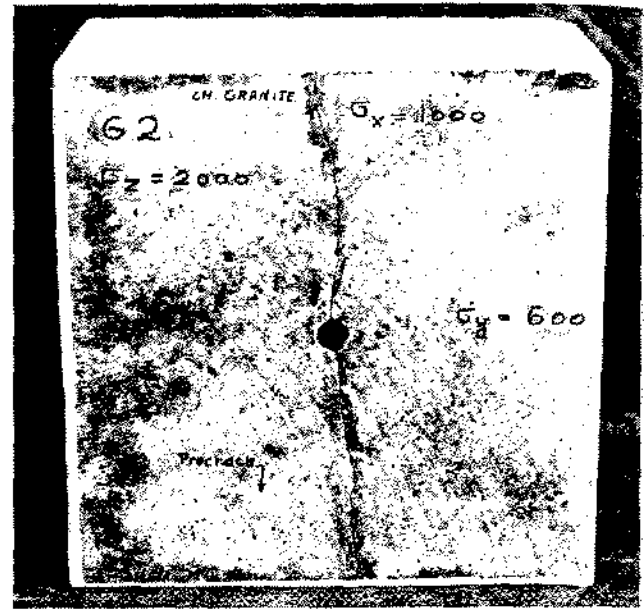


Figure 3. Vertical fracture in Charcoal Granite with precrack.

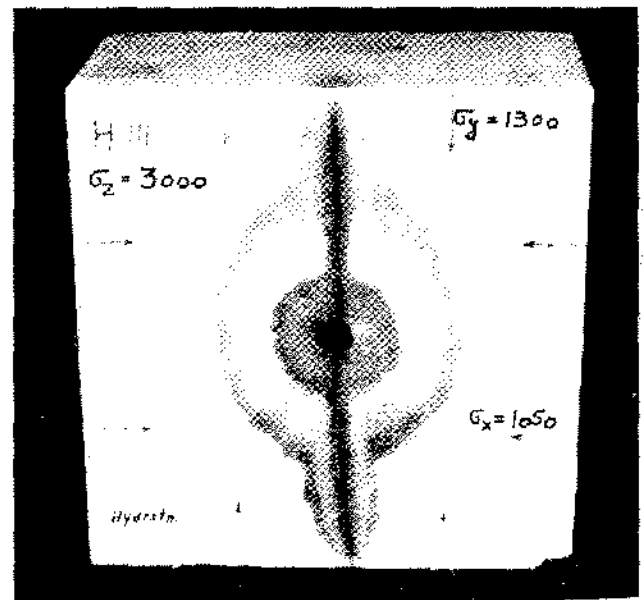


Figure 4. Vertical fracture in hydrostone, also showing the amount of fluid penetration.

into the sample did not affect the predicted direction of the fracture.

3. When the horizontal loading was hydrostatic, the direction of the vertical fracture was at

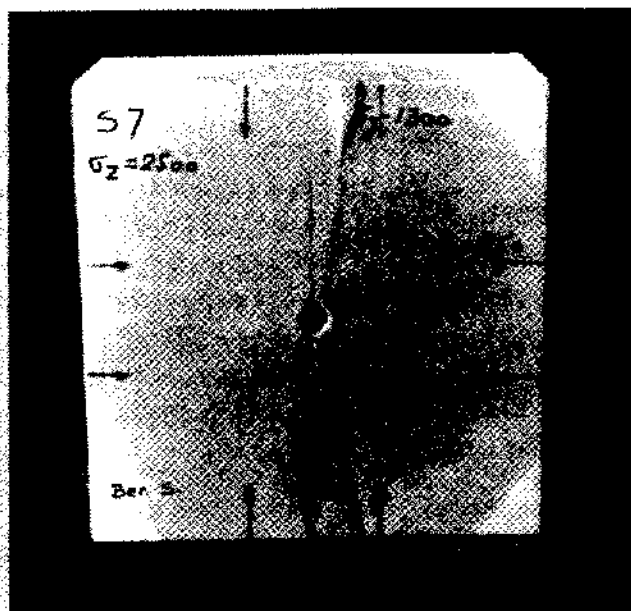


Figure 5. Vertical fracture in highly permeable Berea Sandstone.

random (Fig. 6) and sometimes more than two fractures were observed (Figs. 7 and 8). In a number of samples, vertical notches were induced in the simulated wellbores prior to the fracturing tests, by use of a hydraulic

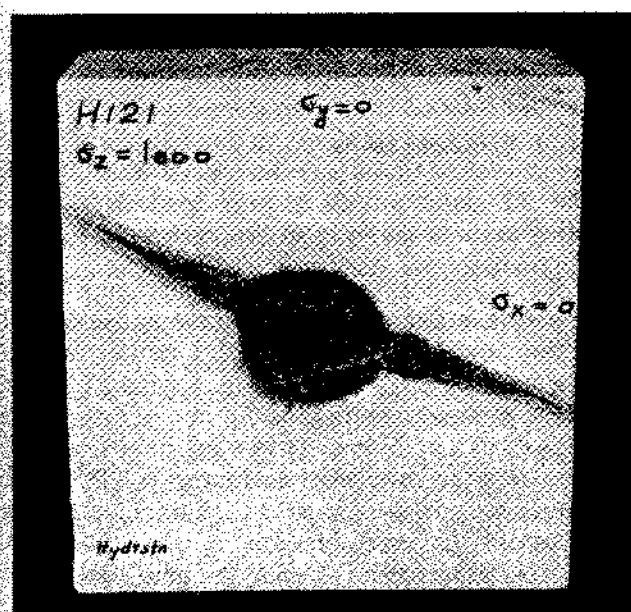


Figure 6. Vertical fracture at random in hydrostone under horizontal hydrostatic stress condition.

jetting technique. With no horizontal loading, the tips of these notches provided the weakest points around the hole and all fractures initiated there and extended in the general

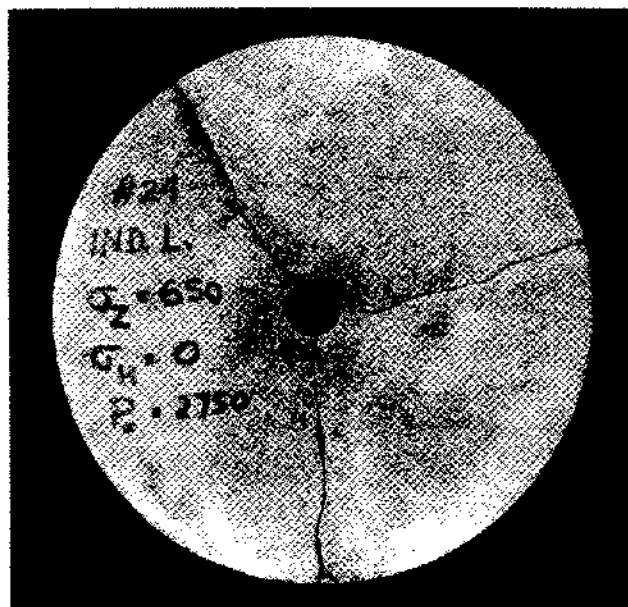


Figure 7. Three evenly distributed vertical fractures in Indiana Limestone under no horizontal loading.

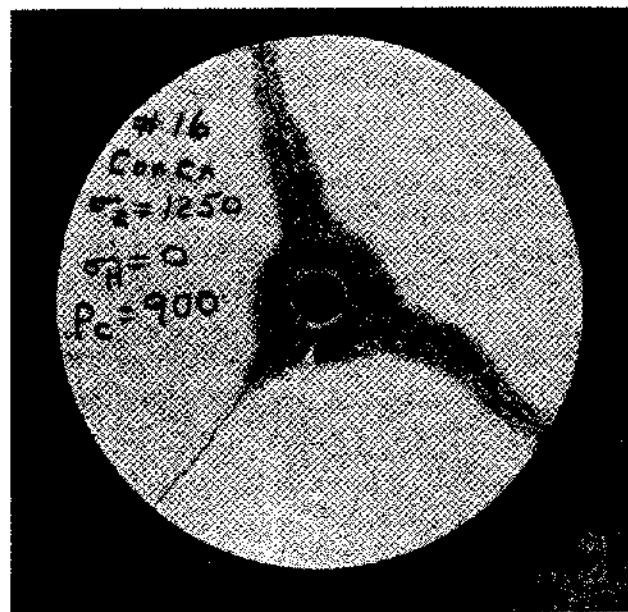


Figure 8. Three evenly distributed vertical fractures in Cordova Cream under no horizontal loading.

direction of the notch. Figures 9 and 10 show typical fractures in notched wellbores. The horizontally sectioned samples of Ohio Sandstone and Cordova Cream, respectively,

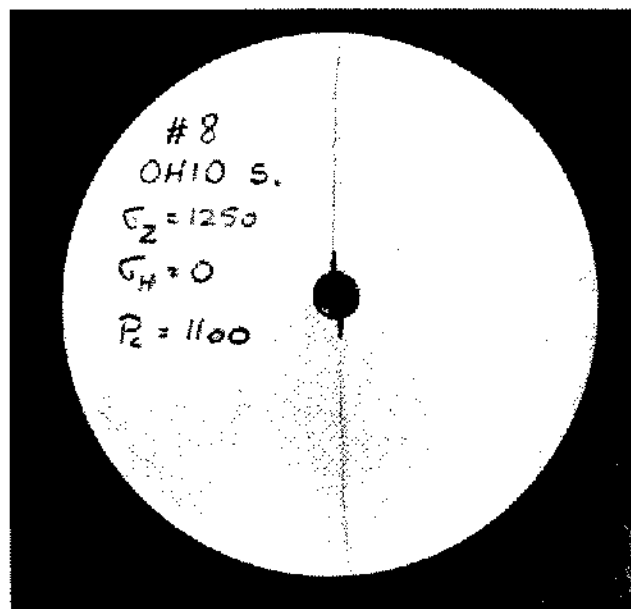


Figure 9. Hydraulic fracture in vertically pre-notched Ohio Sandstone.

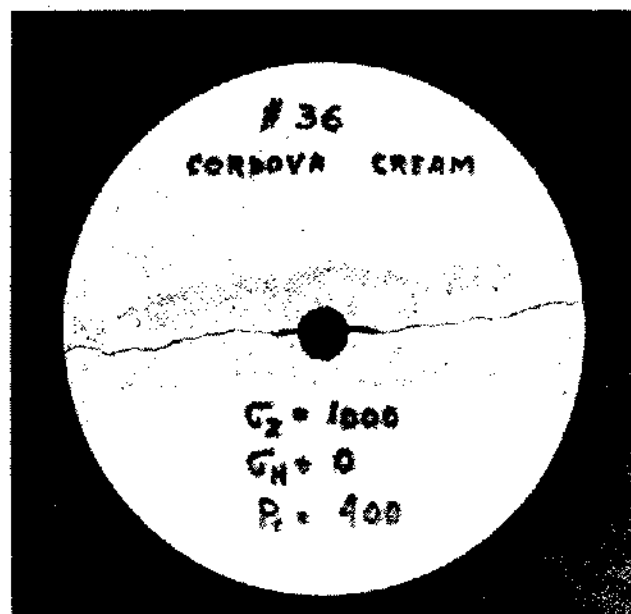


Figure 10. Hydraulic fracture in vertically pre-notched Cordova Cream.

exhibit vertical fractures that were affected by the presence of notches. The latter not only controlled the direction of induced fractures, but also lowered considerably the breakdown pressure. For example, in Ohio Sandstone the  $P_c$  decreased from 1800 psi to 1100 psi, and in Cordova Cream from 1050 psi to 400 psi.

4. In unnotched samples, the pressure required to initiate vertical fractures was close to that predicted by equations (2). Figure 11 shows

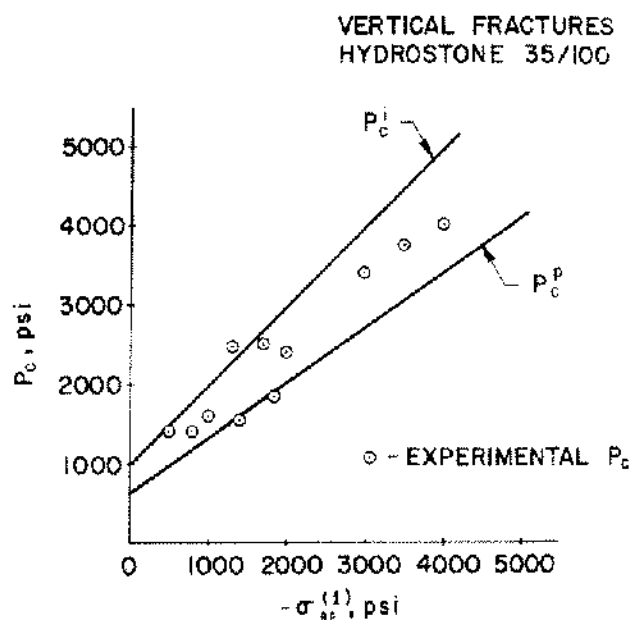


Figure 11. Relationship between theoretically predicted  $P_c^i$  and experimental values of breakdown pressure in impermeable Charcoal Granite.

the relationship between the experimental points and the theoretical curve in the case of the impermeable Charcoal Granite.  $\sigma_{\theta\theta}$  is given by  $\sigma_{\theta\theta} = 3\sigma_{22} - \sigma_{11}$ . Figure 12 shows the same relationship in the case of permeable hydrostone. The experimental points are not as close to the theoretical curve for  $P_c^p$  as the granite points are to  $P_c^i$  (Fig. 11), but it should be remembered that two more rock parameters are involved in the permeable case. Hence the predictions are not as accurate.

5. In a number of tests, two vertical holes were drilled in 5.0 inch diameter cylindrical

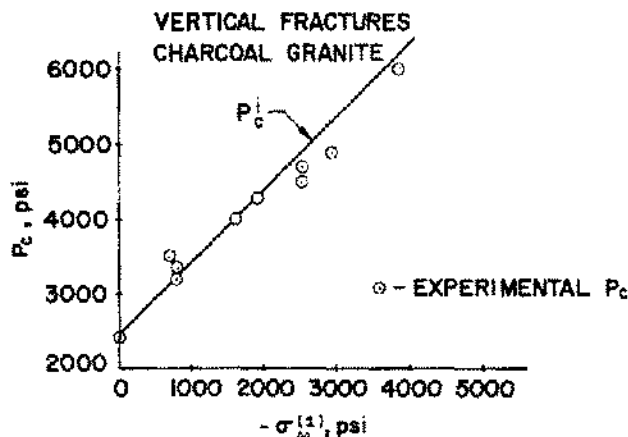


Figure 12. Relationship between theoretically predicted  $P_c^i$  and  $P_c^e$  and experimental values of breakdown pressure in hydrostone 35/100 (35 parts hydrostone to 100 parts water by weight).

samples, to simulate an injection-production set of wells. The distance between the holes was eight times the hole diameter. In these tests no horizontal loading was applied, and the average vertical load was 500 psi. The simultaneous pressurization of both holes resulted in a vertical fracture that emanated from one hole and did not necessarily extend in the direction of the other (Fig. 13). The separate pressurization of each hole yielded fractures that extended at random (Fig. 14). However when vertical notches were induced in the simulated wellbores, the chances of connecting the holes through fracturing were vastly increased. Figure 15 shows a horizontal section of an Indiana Limestone sample in which one of the holes had been vertically notched prior to its fracturing. The notch, as observed, had been directed towards the other well and its direction was followed by the fracture. When the other well was then pressurized, the resulting fracture easily linked to the former. Note the lower breakdown pressure required in the notched hole. Figure 16 depicts another method designed to eliminate guesswork from communicating. Here both wells had been vertically notched in mutually perpendicular planes. The hydraulic fractures joined at some short distance from the production well. This method of double-notching is especially recommended for the field where one cannot expect a hydraulic fracture to extend in a perfect straight line. With two

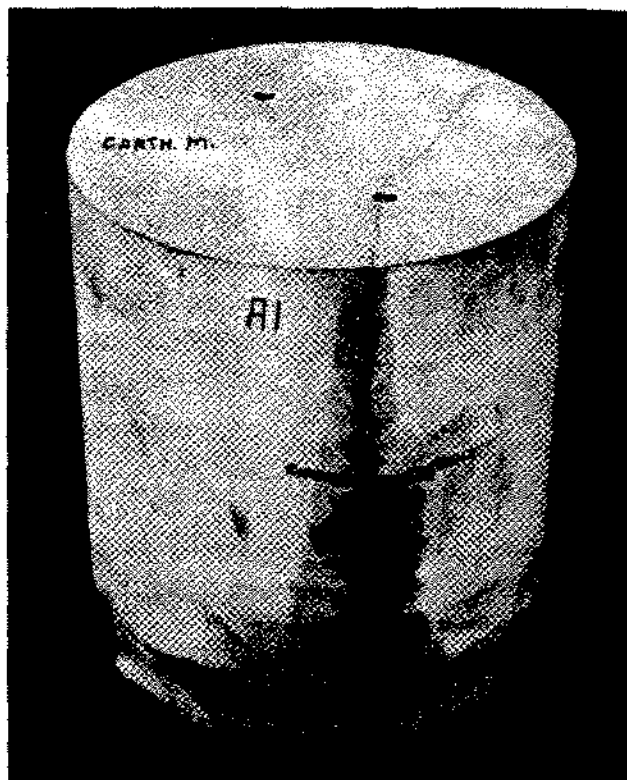


Figure 13. Vertical fracture in Carthage Marble caused by simultaneous pressurization of both wells.

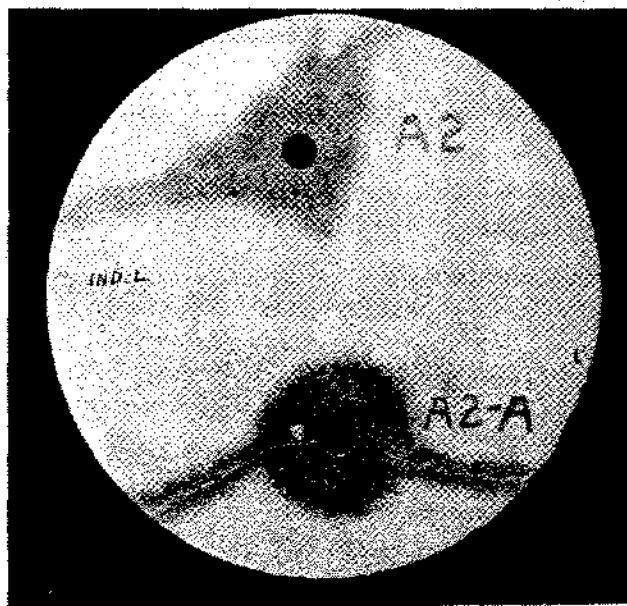


Figure 14. Vertical fractures in Indiana Limestone obtained by separate fracturing of both wells.



perpendicular fractures, they are always bound to meet at some distance from the target well.

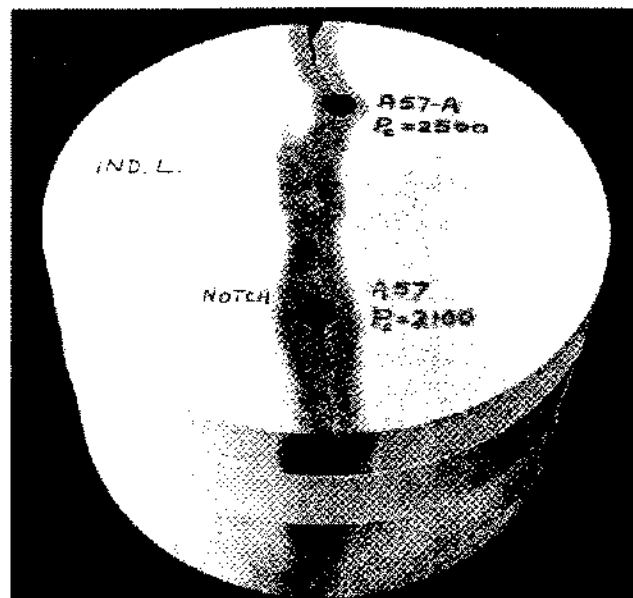


Figure 15. Vertical fractures in Indiana Limestone, obtained by separate fracturing of the wells, with one well being vertically pre-notched in the direction of the other.

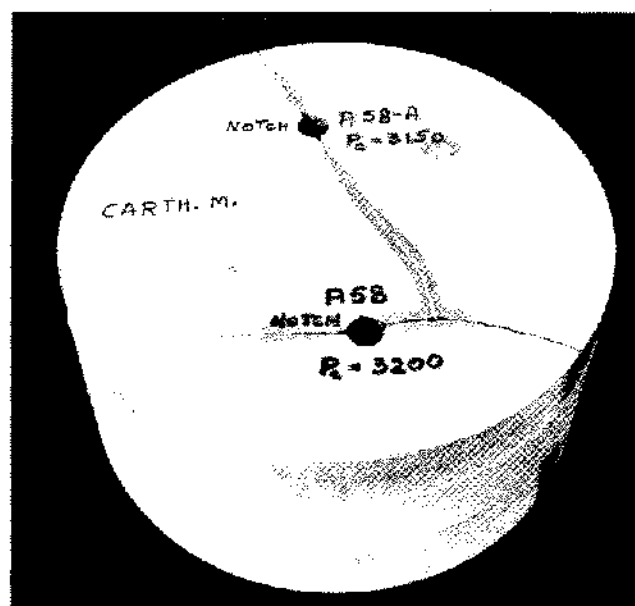


Figure 16. Vertical fractures in Carthage Marble where both wells were pre-notched in mutually perpendicular planes.

## FIELD TESTS

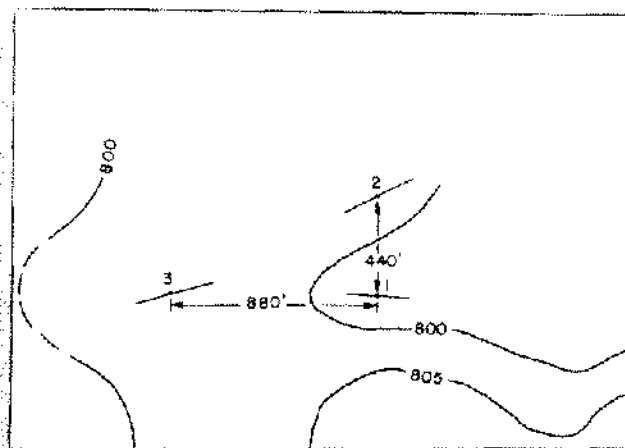
One of the main obstacles to a better understanding of the hydraulic fracturing phenomenon is the great difficulties encountered in conducting scientific testing in the field. A fracturing job is conducted by remote control, from the surface, and there is no access to the pay formation for the purpose of verifying the direction and size of the fracture. Many researchers have suggested the use of different geological phenomenon as an indication of vertical fracture orientation. Occurrence of normal faults (Hubbert, 1957), regional dip in formations (Frazer, 1962), strike of surface joints (Overby, 1968) have all been theoretically correlated to orientation of vertical fractures. In areas where strong evidence of faulting exists or where a surface joint survey is done, the approximate direction of hydraulic fracturing can possibly be predicted. However, a proven testing tool for verifying fracture azimuth at the wellbore, notwithstanding the availability of geological data, is the oriented impression packer.

Such packers were used in the field tests described below. They consisted of a replaceable rubber sleeve, 10 or 20 feet long, mounted on an aluminum mandrel. The sleeve was made of cured reinforced rubber coated on the outside with a layer of uncured rubber. The lower portion of the mandrel contained a pressure relief valve and a landing seat for orientation of a compass running case. The packer was lowered on tubing into the well to the interval under investigation and then the rubber sleeve was hydraulically inflated. Perforations in the mandrel permitted an even distribution of pressure within the packer. The packer was inflated until full contact with the wellbore wall was achieved. The pressure in the packer was maintained for about one hour at a maximum of 300 psi above the reservoir pressure. This allowed the uncured rubber of the sleeve to expand and conform to the wellbore wall, while a magnetic compass was used to determine the orientation of the tool. The compass was lowered into the well after the packer was inflated and fixed in its landing seat at the bottom of the mandrel. Multiple compass pictures were taken from a camera located in the running case, and where possible two separate compasses were used on each test. At the conclusion of the test the pressure relief valve was opened and the impression packer removed from the well. From the location of the machined groove on the compass landing seat and the photographs during the test, the magnetic north on the



packer was established. Thus, the orientation of any fractures or other irregularities, recorded permanently on the impression packer, could be easily determined.

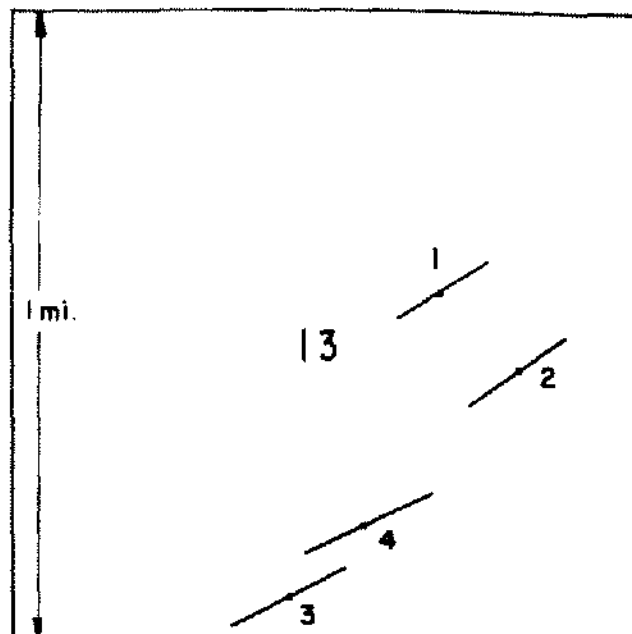
Studies using impression packers were conducted in New York, Ohio and Illinois. The purpose of these studies was to determine the type and orientation of hydraulically induced fractures in oil producing formations. The breakdown and instantaneous shut-in pressures were also recorded. The reservoir studied in New York was the Richburg Oil Sand in Alma Township, Allegany County. In Ohio, the study included the Clinton Sandstone in Falls Township, Hocking County, and in Illinois, a carbonate reservoir was investigated. Locations of the tested wells in two of the areas are shown in Figures 17 and 18.



LOT 91  
ALMA TOWNSHIP  
ALLEGANY COUNTY, NEW YORK

Figure 17. The distribution of the treated wells, and the direction of vertical fractures in the New York field.

The impression packer results show that fractures created in each of the wells were vertical over the major portion of the treatment interval (Fig. 19). The average azimuth of these fractures in each well is given in Table 1 and shown diagrammatically (except for Illinois) in Figures 17 and 18. Table 2 enumerates some of the physical properties of the formations. It can be easily verified that in each of the three fields the induced fractures were



SECTION 13  
FALLS TOWNSHIP  
HOCKING COUNTY, OHIO

Figure 18. The distribution of the treated wells, and the direction of vertical fractures in the Ohio field.

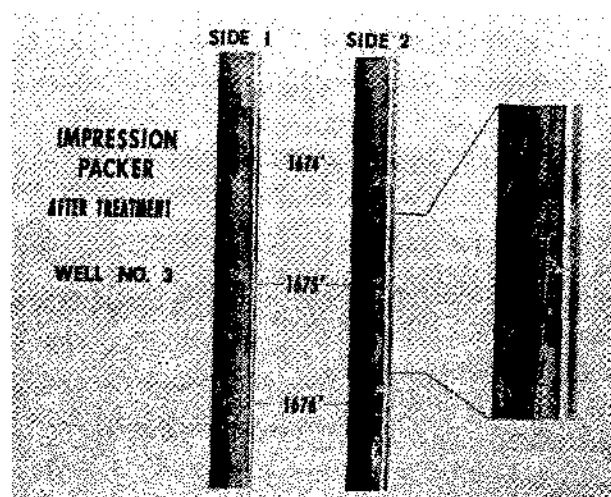


Figure 19. Packer with typical vertical fracture impression from well No. 3 in New York.

TABLE 1. Hydraulic Fracturing Results in Three Field Tests

Location	Well No.	Depth Feet	Critical Breakdown Pressure $P_c$ , psi	Instantaneous Shut-in Pressure $P_s$ , psi	Fracture Azimuth East of True North
New York	1	1607-16	5264*	1864	93°
	2	1677-82	5848*	2013	65°
	3	1671-79	3256	2161	74°
Ohio	1	2622-32	2938	2238	62°
	2	2634-52	-	-	59°
	3	2671-81	2934	2259	67°
	4	2662-71	3054	2154	68°
Illinois	1	314-332	539	-	49°
	2	321-338	643	393	67°
	3	314-323	588	338	72°
	4	310-327	738	338	58°
	5	298-318	733	333	66°

\* Recently completed cable tool hole. The mud, still lining the hole, accounts for the unusually high values of breakdown pressure.

TABLE 2. Physical Properties of Three Formations Tested

Location	Tensile Strength psi	Porosity %	Permeability md.	Poisson's Ratio	Reservoir Pressure psi
New York	575	13.5	0.5	0.1	530
Ohio	1000	15.0	33.0	0.2	600
Illinois	725	22.0	8.0	0.2	0

nearly parallel to each other. Table 1 also gives the breakdown (critical) pressures and the instantaneous shut-in pressures in the fractured wells. Again, a striking closeness between the values of the fracturing pressures in each of the fields can be observed. Based on our hypothesis relating in situ stresses to hydraulic fracturing pressures and directions, it appears that in each of the studied formations there was one constant tectonic stress field. By hydraulically fracturing a sample of wells like any of the three samples mentioned in this paper, one could get enough information to help design more scientifically the layout of a newly prospected field, or determine whether hydraulic fracturing may be feasible in an established producing field.

### CONCLUSIONS

The present report is merely an attempt to improve the existing knowledge of vertical hydraulic fractures. Theoretically it is shown that the breakdown and shut-in pressures, usually recorded during a fracturing job, as well as the direction of fracture, are directly related to the principal tectonic stresses that exist in the formation. Indeed, laboratory tests on simulated wellbores indicate that the theoretically predicted breakdown pressures, in both porous and non-porous rock, were close to the experimental results. Moreover, as theoretically expected, vertical fractures were always tensile ruptures that initiated and extended in a plane perpendicular to the direction of the smaller simulated horizontal compressive principal tectonic stress. In samples where the simulated horizontal in situ stress condition was hydrostatic vertical fractures extended at random and sometimes three rather than two ruptures were obtained. Notching appeared to be a helpful tool in controlling direction of fractures in such hydrostatic cases and was instrumental in achieving communication between two wells. The three field tests reported here merely recorded the breakdown and shut-in pressures during hydraulic fracturing operations, and the azimuth at the wellbores of resulting vertical fractures from impression packer readings. The closeness between the pressures and directions in each of the fields leaves little doubt as to the relationship between fracturing and tectonic stresses. Each of the three groups of wells belong to the same production field in a rather uniform geological system. Hence there is no reason to expect that tectonic stresses would vary considerably from one well to another. The field results verify

this assumption. It seems that a sample of wells, intelligently picked and hydraulically ruptured, could provide with the necessary information about direction of fractures, breakdown pressures, and communication possibilities in an entire field. If results in the sample are hardly uniform as far as fracture direction, it is probably because of the tectonic stresses in the horizontal plane being hydrostatic. In such a case, vertical notching of wells may prove very beneficial.

### NOMENCLATURE

$P$	=	fluid pressure
$P_o$	=	reservoir pore fluid pressure
$P_c$	=	breakdown (critical) pressure
$P_c^i$	=	breakdown pressure in impermeable rock
$P_c^p$	=	breakdown pressure in permeable rock
$P_s$	=	wellbore instantaneous shut-in pressure
$P_w$	=	wellbore pressure prior to fracturing
$S_{ij}$	=	stress tensor
$S_{11}$	=	smaller horizontal principal tectonic stress (tension taken as positive)
$S_{22}$	=	larger horizontal principal tectonic stress
$S_{33}$	=	vertical principal tectonic stress
$\alpha$	=	parameter of a porous material
$\nu$	=	Poisson's ratio
$\sigma_{ij}$	=	effective stress tensor
$\sigma_{ij}$	=	effective stress tensor around the wellbore due to tectonic stresses
$\sigma_x, \sigma_y, \sigma_z$	=	compressive loads applied to specimens
$\sigma_t$	=	tensile strength in the horizontal plane as applied to hydraulic fracturing.

### ACKNOWLEDGEMENTS

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### REFERENCES

- Anderson, E.M., 1963, *The dynamics of faulting*: Edinburgh and London, Oliver and Boyd, 206 p.
- Frazer, C.D., and Pettitt, B.E., 1962, Results of a field test to determine the type and orientation of hydraulically induced formation fracture; *Jour. Petr. Tech.* v. 14, p. 463.
- Haimson, B., 1968, Hydraulic fracturing in porous and non-porous rock and its potential for determining in-situ stresses at great depth: Ph.D. Thesis, University of Minnesota.
- Haimson, B., and Fairhurst, C., 1967, Initiation and extension of hydraulic fractures in rocks: *Soc. Petr. Eng. Jour.*, Sept. 1967, p. 310-318.
- , 1969, Hydraulic fracturing in porous-permeable materials: *Jour. Petr. Tech.* (to be published).
- Hubbert, M.K., and Willis, D.G., 1957, Mechanics of hydraulic fracturing: *Trans. AIME.* v. 210, p. 153.
- Mann, R.L., and Fatt, I., 1960, Effects of pore fluids in the elastic properties of sandstone: *Geophysics*, v. 25, p. 433.
- Overbey, W.K., and Rough, R.L., 1968, Surface joint patterns predict wellbore fracture orientation: *Oil and Gas Jour.*, Feb. 26, p. 84.